



## Numerical radius: New Extensions and Inequalities

Ali Zamani\*

Department of Mathematics, Farhangian University, Tehran, Iran

---

### Abstract

We firstly define a norm on the space of bounded linear operators on a Hilbert space, which generalizes the numerical radius norm. We investigate basic properties of this norm and prove inequalities involving it. Further, for a positive element  $a$  in a unital  $C^*$ -algebra  $\mathfrak{A}$  we define a semi-norm on  $\mathfrak{A}$ , which generalizes the  $a$ -operator semi-norm and the  $a$ -numerical radius. We derive new upper and lower bounds for the  $a$ -numerical radii of elements in  $\mathfrak{A}$ . Some applications and other related results are also discussed.

**Keywords:** Numerical range, numerical radius, inequality.

**Mathematics Subject Classification [2010]:** 47A12, 47A30, 46L05.

---

---

\*Email address: zamani.ali85@yahoo.com