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Stationary Graph Signals

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Abstract

While conventional discrete signals are represented over grids, we currently deal with a number of signal types for which no well-defined grid is applicable; data related to social networks is among the examples. An alternative way for representing such signals is to assume a graph, where each node plays the role of a grid point. In other words, each node contains a part of the whole signal and based on the connections in the graph, these parts could be thought of as related to each other. In contrast to the conventional 1D discrete signals where each signal tap is adjacent to its predecessor and successor taps, in graph signals, the adjacency of a signal tap is not necessarily limited to 2 other taps. Obviously, there are more degrees of freedom in graph signals, which makes them a more versatile modeling platform. The downside is that the processing techniques which are well-studied for decades for conventional discrete signals shall be revisited and redefined. As we will see in this talk, some of the equivalent processes and definitions in the graph signal domain are quite non-trivial.

Keywords: Discrete signals, Fourier transform, frequency domain, and graphs.
Mathematics Subject Classification [2010]: 15A06, 15A18, 15A20.

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1 Introduction

For a graph signal, we first need a graph $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2 \dots, v_n\}$ denotes the set of vertices and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ stands for the set of edges. In general, the edges could be directed, which implies that for some $v_i, v_j \in \mathcal{V}$ we might have $(v_i, v_j) \in \mathcal{E}$ but $(v_j, v_i) \notin \mathcal{E}$. Besides, the graph is equipped with a weight matrix $\mathbf{W}_{n\times n}$, which is very similar to the adjacency matrix except that the non-zero elements are not necessarily 1:

$$i, j \in \{1, 2 \dots, n\} : \quad (\mathbf{W})_{i,j} = \begin{cases} w_{i,j} \geq 0, & (v_i, v_j) \in \mathcal{E}, \\ 0, & (v_i, v_j) \notin \mathcal{E}. \end{cases}$$

The weight matrix actually encodes the connectivity of the graph. Edges with larger weights are assumed to be more strongly connected.

Now that we introduced the graph, we need to define the graph signal $\mathbf{x}_{n\times 1}$, which takes a scalar value x_i at each vertex v_i. Although we express the signal graph using a vector, the order of elements in this vector depend on the labeling of the graph vertices. As this labeling does not affect the structure and properties of the graph, we need to

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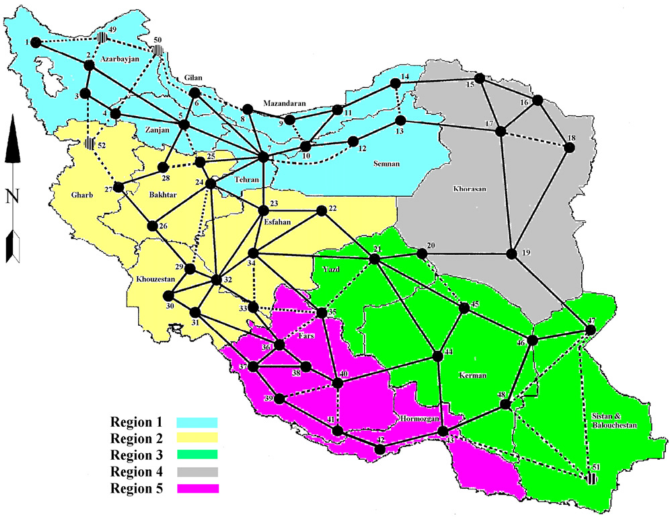

Figure 1: Iran's power distribution grid for 400KV lines [1, 2].

devise signal processing tools and techniques that commute with reordering of the graph signal.

To better illustrate the concept of graph signals, let us consider Iran's power distribution grid for 400KV lines in Figure 1. Each node in this graph is either connected to a power generator, or a cluster of power consumers (after reducing the voltage in a number of steps). The connection between the nodes correspond to the existence of physical power lines in between. Here, the edge weights could be defined as the inverse of the effective total resistance between the nodes. In simple words, nodes that are physically far apart are expected to have small edge weights; similarly, nodes that are not connected can be interpreted as being connected by an infinite-length power line.

A simple example of a graph signal here is the voltage, i.e., at vertex v_i the value x_i of the graph signal is given as the measured voltage of the substation at a given time instance. Ideally, this value should be 400KV equally at all substations; however, due to line losses, we observe voltage drops. Based on the definition of the graph and the electrical properties, the graph signal values at neighboring vertices that are connected with strong edges shall be almost equal. This suggests that the graph signal is almost smooth on the graph; in other words, when we move along the edges, signal values do not change drastically. This behavior resembles the lowpass nature of conventional graph signals.

In this talk, upon simple graph operators such as the *shift*, we build graph Fourier transform (GFT) and interpret classical concepts such as the frequency domain, lowpass property and etc based on GFT. Next, we consider stochastic signals over graphs and investigate the notion of stationarity. The basics of graph signal processing reviewed in this talk are taken from [3] and [4].

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